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Oded Stark, Doris A. Behrens

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**In search of an evolutionary edge:
trading with a few, more, or many**

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Zentrum für Entwicklungsforschung (ZEF)
Center for Development Research
Walter-Flex-Strasse 3
D – 53113 Bonn
Germany
Phone: +49-228-73-1861
Fax: +49-228-73-1869
E-Mail: zef@uni-bonn.de
<http://www.zef.de>

The authors:

Oded Stark, Universities of Bonn, Klagenfurt, and Vienna; Warsaw University; Warsaw School of Economics. Contact: ostark@uni-bonn.de

Doris A. Behrens, University of Klagenfurt; Vienna University of Technology. Contact: doris.behrens@uni-klu.ac.at

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Abstract

Consider a population of farmers who live around a lake. Each farmer engages in trade with his m adjacent neighbors, where m is termed the “span of interaction.” Trade is governed by a prisoner’s dilemma “rule of engagement.” A farmer’s payoff is the sum of the payoffs from the m prisoner’s dilemma games played with his $m/2$ neighbors to the left, and with his $m/2$ neighbors to the right. When a farmer dies, his son takes over. The son who adheres to his father’s span of interaction decides whether to cooperate or defect by considering the actions taken and the payoffs received by the most prosperous member of the group comprising his father and his father’s m trading partners. Under a conventional structure of payoffs, it is shown that a large span of interaction is detrimental to the long-run coexistence of cooperation and defection, and conditions are provided under which the social outcome associated with the expansion of trade when individuals trade with a few is better than that when they trade with many. Under the stipulated conditions it is shown, by means of a static comparative analysis of the steady state configurations of the farmer population, that an expansion of the market can be beneficial in one context, detrimental in another.

Kurzfassung

Wir betrachten eine Population von Farmern, die rings um einen See leben. Jeder Farmer betreibt mit seinen m nächsten Nachbarn Tauschhandel, wobei m als „Interaktionsspanne“ bezeichnet wird. Der Handel folgt den „Spielregeln“ eines Gefangenendilemmas. Das Einkommen eines Farmers ergibt sich als Summe der Auszahlungen der m einzelnen „Spiele“, die der Farmer mit seinen $m/2$ Nachbarn zur Linken und $m/2$ Nachbarn zur Rechten spielt. Stirbt ein Farmer, übernimmt sein Sohn die Farm. Der Sohn, der die Interaktionsspanne m des Vaters beibehält, entscheidet sich für oder gegen die Wahl der kooperativen Strategie, indem er das Verhalten des wohlhabendsten Farmers aus einer Gruppe von Vorfahren repliziert, die aus seinem Vater und dessen m Handelspartnern besteht. Für eine konventionelle Auszahlungsstruktur wird gezeigt, dass eine große Interaktionsspanne der nachhaltigen Koexistenz von kooperativem und nicht-kooperativem Verhalten abträglich ist. Weiters werden Bedingungen für die Auszahlungsstruktur dargelegt, unter denen das durch eine Ausweitung der Handelsbeziehungen generierte Pro-Kopf-Einkommen höher ist, wenn Individuen ursprünglich wenige Handelspartner hatten, als wenn Individuen ursprünglich sehr viele Handelspartner hatten. Somit wird bei entsprechender Auszahlungsstruktur mittels statisch komparativer Analyse der Gleichgewichtskonfiguration der Farmer-Population nachgewiesen, dass eine Markterweiterung in manchen Fällen vorteilhaft sein kann, in anderen aber nicht.

1 Introduction

The evolution of cooperating behavior in a population can be analyzed by drawing upon an iterated prisoner's dilemma game (cf. Bergstrom and Stark, 1993). Within such a framework, the conditions for cooperators not to be eliminated by natural selection have to be such that some sort of "preferential interaction" occurs, for example when interaction is confined to homogenous sets, or clusters, of cooperators.

The formation of cooperator clusters in a spatial layout of the iterated prisoner's dilemma permits the long-run coexistence of cooperating and non-cooperating behavior if (in terms of payoffs) cooperators in the interior of a cluster do well as compared with defectors at the boundary of the cluster (cf. Nowak and May, 1993). Bergstrom and Stark (1993) identify evolutionary environments that are conducive to the long-run survival of a cooperating strategy: assuming that individuals who live along a circle (around a lake) trade with their adjacent neighbors, and that the descendents of the individuals imitate the strategy of whoever is the most successful - their father or one of his two or four adjacent neighbors, Bergstrom and Stark (1993) concluded that "provincialism may promote cooperation" (p. 153). In such a prisoner's dilemma setting, trade and information do not "behave" symmetrically: trading with others involves collecting information about them, yet information about them can be obtained without trading with them (refer, for example, to Hoffmann, 1999; Alós-Ferrer and Weidenholzer, 2008; Mengel, 2009). In other words, whereas a trade inherently conveys information about the trading, securing information about a potential trading partner does not require trading with him. Stark and Behrens (2010) exploited this asymmetry between trade and information to come up with an extension of the Bergstrom and Stark model and thereby to provide a rigorous support for the Bergstrom and Stark's assertion that "local information" is conducive to the long-run survival of the cooperative strategy: holding the number of the trading partners constant, Stark and Behrens (2010) permitted the size of the set of those from whom individuals learn, "the span of information," to vary and showed how variation in this span impinges on the community's wellbeing.

In this paper we take a step further: we investigate the "fate" of the cooperating strategy and the associated welfare implications when we vary the number of adjacent neighbors with whom individuals trade, termed "the span of interaction," and whose good example descendents mimic. The rule that governs the evolution of the farmer community over time and across

generations could be characterized as an “imitate-the-best” type of behavior.¹ To see why and when trading with many rather than with a few is not conducive to social welfare, it is helpful to reiterate how what we do in this paper relates to Stark and Behrens (2010). In the current paper, the span of information is a close shadow of the span of interaction; the former tracks the latter perfectly (and fully). The asymmetry between the two spans was at the heart of Stark and Behrens (2010), in which the span of information was allowed not to be constrained by the span of trade. What is intriguing is that in both Stark and Behrens (2010) and in the current paper, both spans are found to support cooperation when extended somewhat, viz. “locally,” but are hostile to cooperation when expanded further a field, viz. “globally.”²

The current paper relates to, but takes a different track than a number of closely related writings that also study *cycle structures*. We complement Ellison (1993) by treating imitation as the driving force behind natural selection rather than to a best-reply response. We differ from Eshel et al. (1998) by investigating arbitrary spans of interaction rather than interaction confined to adjacent neighbors (a pattern borrowed from Bergstrom and Stark, 1993), and by analyzing comprehensively the relationship between the span of interaction and the community’s welfare. The stability concept³ and the analysis of the welfare implications distinguish our current paper from work that explicitly builds on Eshel et al. (1998) and implicitly on Bergstrom and Stark (1993), such as Mengel (2009). We also diverge from Ohtsuki and Nowak (2006) by analyzing deterministic dynamics rather than stochastic evolutionary games. And, as already noted, the current offering is distinct from what Stark and Behrens (2010) have done in that here we vary the span of interaction (coinciding with the span of information) rather than vary the span of information alone. Essentially, our current paper differs from the work of others (which, in the main, deals with the theme of the evolutionary stability of cooperation) by including a measurement of the community’s wellbeing. Thus, the current paper complements the received literature in that it studies welfare implications; and, in particular, shows that depending on the relative size of the payoffs, the welfare gains from expanded trade are conditional on the initial span of trade, which in turn is in variance with a basic tenet of international economics that hails market expansion. This result is due to the fact that, under the stipulated conditions, when trade

¹ Alós-Ferrer and Weidenholzer (2008, p. 256) eloquently provide a reasoning for the “tendency [of humans] to focus on salient outcomes, e.g. those leading to high payoffs,” rather than on average outcomes.

² The distinction between “local” and “global” trade aligns with the writings of others (for example, Hoffmann, 1999). A more formal definition in terms of a relation between the size of the population and the number of those with whom individuals trade is given in section 4.

³ We employ the terminology *persistent coexistence of cooperation and defection behavior*. Persistence applies when after the appearance of a defector in a pure cooperator population and following a brief initial phase, the fraction of cooperators remains invariant in time, that is, it stays intact in all the generations that follow.

is “local,” the expansion of trade is conducive to the prevalence and sustainability of cooperation, while when trade is “global,” the expansion of trade is detrimental to the prevalence and sustainability of cooperation.

2 The model

We consider a farmer community consisting of $n \in \mathbb{N}$ individuals who live around a lake, such that each individual has exactly two adjacent neighbors. Without loss of generality, we assume that n is an even natural number. To sustain life, trading with others is mandatory: farmers need to engage in barter, say, in labor inputs or in produce, collaborate (join forces) in production-related activities such as pest control, and so on.

We assume that each individual exchanges goods and services with his m nearest neighbors (that is, with $m/2$ neighbors to the left, and with $m/2$ neighbors to the right) where m is an even natural number.⁴ Thus, $2 \leq m \leq n-2$. We refer to the size of this set of neighbors, which can vary, as the farmer's "span of trade" or "span of interaction." Trade is governed by a prisoner's dilemma "rule of engagement," and each individual's income is the sum of the total payoffs from the m prisoner's dilemma games, where the payoff matrix of a single game is given by

		Column player	
		<i>C</i>	<i>D</i>
R ow player	<i>C</i>	<i>R, R</i>	<i>S, T</i>
	<i>D</i>	<i>T, S</i>	<i>P, P</i>

and where $0 \leq S < P < R < T$.⁵

⁴ We concentrate on the influence of the m adjacent neighbors on the decision to act cooperatively. We do so not because we believe that these are the only neighbors who matter, but rather as a heuristic device aimed at embracing the idea that neighbors nearby matter more (influence decisions more) than distant neighbors, which in turn draws on the premise that, in some sense, information decays with distance. In the real, and of long-duration world, a reason why farmers imitate farmers nearby more than farmers far away may well be that neighborliness is the outcome of selection, and that in terms of some unobserved characteristics, adjacent farmers are more alike than randomly selected farmers, which in turn could explain why rather than because of mere geographical proximity, the behavior and actions of farmers nearby are of more relevance for emulation than the behavior and actions of farmers farther away.

⁵ The prisoner's dilemma is a two-person non-zero-sum game originally formulated in 1950 by RAND scientists Merrill Flood and Melvin Dresher (Flood, 1958) in order to illustrate how individuals' optimization can entail outcomes that are socially sub-optimal. Convention has it that in a prisoner's dilemma framework, the letter *C* stands for playing the cooperative strategy, and the letter *D* stands for playing the defection strategy. Consider two farmers who play each *C*. For example, each engages in pest control activities. Consequently, each receives a payoff R (as in "Reward for mutual cooperation"). Consider a cooperator farmer whose neighboring farmer is a defector, that is, a farmer who declines to pursue pest control activities. Then, the cooperator farmer loses some payoff as compared to when his counterpart exercises pest control too, whereas the defector farmer gains: he benefits from (albeit a reduced application of) pest control, but incurs no application cost. The cooperator farmer then receives a payoff $S < R$ (S as in "Sucker's payoff"), whereas the defector farmer reaps a payoff $T > R$, (T as in "Temptation to defect"). When a farmer elects not to engage in pest control, building on the expectation that the other farmer will, none applies, and (by symmetry) each ends up with a payoff of $P < R$ (P as in "Punishment for mutual defection").

Quite naturally, we additionally assume that defectors (playing D) are relatively successful in a mixed population, but less successful than cooperators (playing C) in a pure cooperator community or in a cooperator cluster of sufficient size. For any $2 \leq m \leq n-2$ this assumption translates into

$$(m-1)R + S < \frac{m}{2}(T + P) < mR, \quad (1)$$

guaranteeing that a defector at the border of a cluster of at least $\frac{m}{2}$ defecting farmers (who receives a payoff of $\frac{m}{2}(T + P)$) is worse off than a cooperator surrounded by at least m cooperators (that is, by $\frac{m}{2}$ cooperators to the left, and by $\frac{m}{2}$ cooperators to the right, receiving thereby a payoff of mR), yet is better off than a cooperator who is “ripped off” by (at least) one defector (receiving a payoff of $(m-1)R + S$). Within the present context, the right hand side inequality in (1) is a well-established convention (cf., for example, Bergstrom and Stark, 1993; Stark and Behrens, 2010), guaranteeing that cooperation is efficient in the two-person relationship, since $T + S < T + P$ where, in turn, $T + P$ is smaller than $2R$. The left hand side inequality in (1) is introduced to enhance the analytical tractability of the welfare analysis undertaken in section 4. Our results below regarding the prevalence of cooperation would, however, hold just as well if (1) were to be replaced by $T + P < 2R$.

We introduce dynamics to the farmer community by postulating that when a farmer dies, his son takes over. The son who replicates his father’s span of interaction (m is not a subject of choice) decides whether to play cooperate or to play defect in all his trades (cf., for example, Bergstrom and Stark, 1993, and Stark and Behrens, 2010) by considering the actions taken and the payoffs received by a group comprising his father and the set of the m neighbors that his father traded with.⁶ We introduce optimization (maximization subject to an informational constraint) by postulating that when the farmers’ n descendents take over the farms, they replicate each the behavior of the most prosperous out of their father, their father’s $\frac{m}{2}$ neighbors to the left, and their father’s $\frac{m}{2}$ neighbors to the right.⁷

What happens, however, when an individual’s choice is based on assessment of the payoffs of an increasing number of neighboring individuals (trading partners)? While then the seeming success of a defector (cf. assumption (1)) can spread more widely among a community of (cooperative) farmers, so can the information about successful cooperators (who are cozily nested, and thereby provided with high payoffs in a neighborhood of cooperators). Does the

⁶ The possibility that the set of individuals from whom the son learns is larger than the set consisting of his father and the m trading partners of his father is discussed in Stark and Behrens (2010).

⁷ As a tie-breaking convention, we assume that when the descendents observe at least two forefathers who played different strategies but derived identical payoffs, the behaviour of their own father is imitated.

defection strategy spread then “faster” if the span of interaction is larger? And how is social welfare affected when the number of trading partners increases? In the next section we shed light on the relationship between the span of interaction and the nature of (the intertemporal) equilibrium. In section 4 we assess the impact of the span of interaction on the wellbeing of the community. In section 5 we provide a brief summary of our findings.

3 The long-run composition of the farmer community

Initially, let all the farmers be cooperators playing C , as depicted in Figure 1; the generation 1 fraction of farmers who are cooperators is $x_1 = 1$. In Figures 1, 2, and 3, a letter, C or D , represents a farmer's strategy, and a mathematical expression by the side of the letter delineates the farmer's total payoff (which can be conceived as the output of some agricultural good).

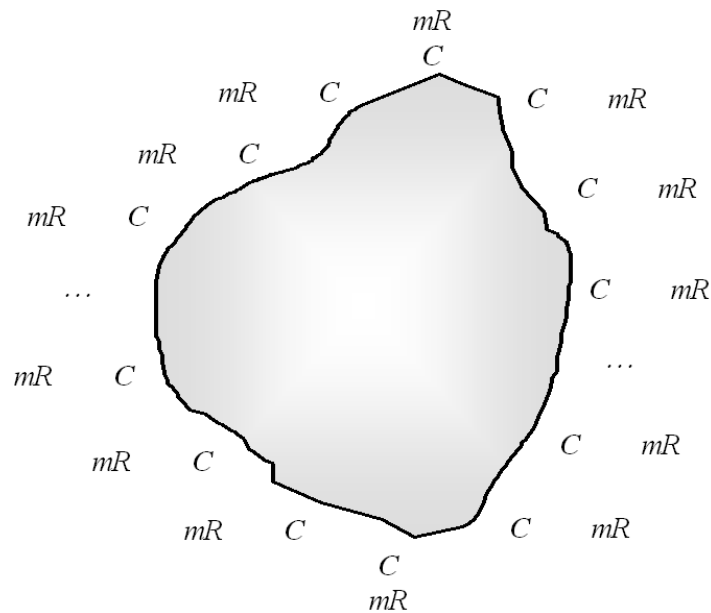


Figure 1: The generation 1 composition of a community of n farmers, for a span of interaction m

Imagine, alternatively, that in the all-cooperator community of at least $m+2$ farmers (that is, when $n \geq m+2 \geq 4$), one of the cooperators mutates to become a defector, as depicted in Figure 2. The fraction of cooperators in the community then changes to

$$x_1 = \frac{n-1}{n} = 1 - \frac{1}{n}. \quad (2)$$

The information about the payoff garnered by the defector in generation 1 (this payoff is mT , see Figure 2) spreads in generation 2 to the descendants of $m/2$ neighbors to the left of the son of

the defector, and to the descendents of $\frac{m}{2}$ neighbors to the right of the son of the defector, where $m = 2, 4, 6, \dots$. Correspondingly, a fraction of cooperators

$$x_2 = x_1 - \frac{m}{n} = 1 - \frac{1}{n}(m+1) \tag{3}$$

survives in generation 2 (see Figure 3).

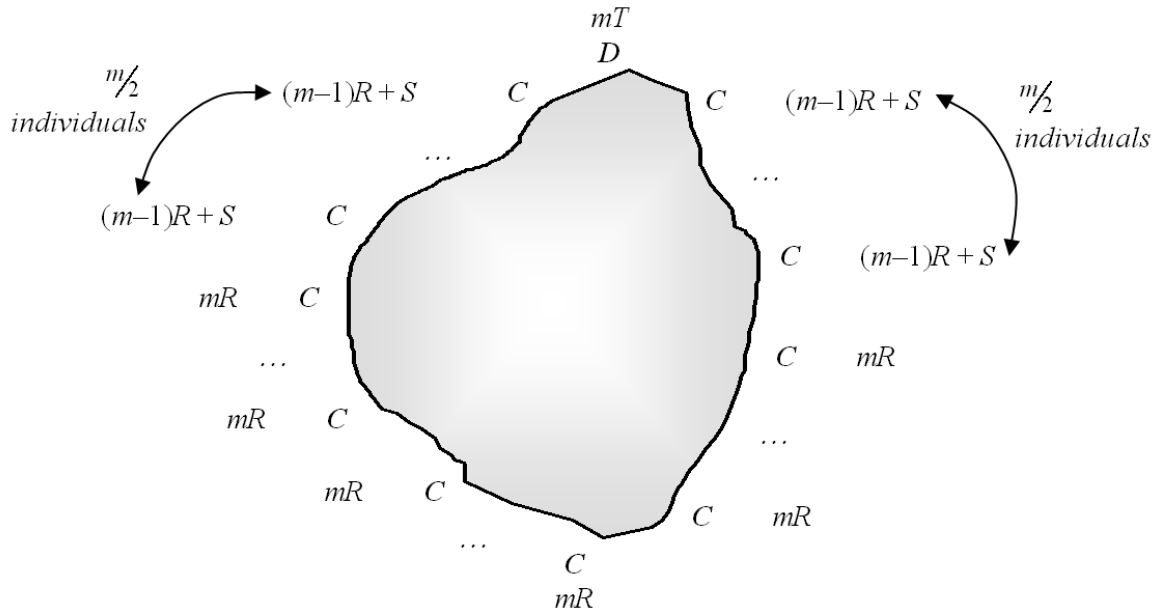


Figure 2: The generation 1 composition of a community of n farmers upon the appearance of a single defector, for a span of interaction m

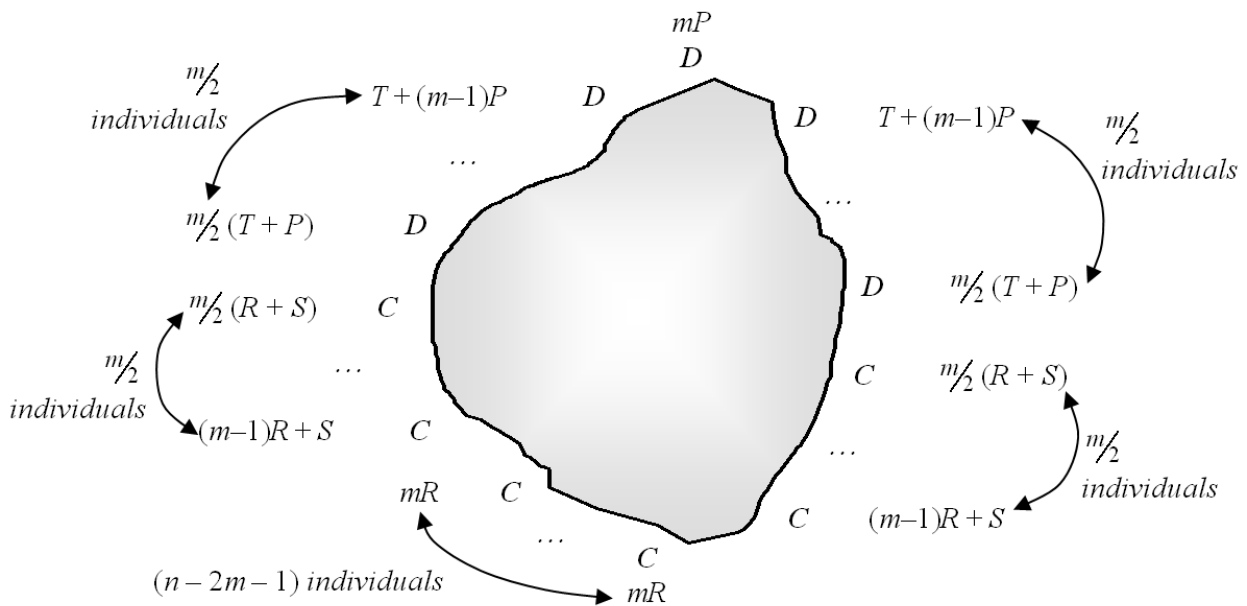


Figure 3: The generation 2 composition of a community of n farmers following the appearance of a single defector, for a span of interaction m

When $n < 2(m+1)$, the third generation (and subsequently all future generations) will consist solely of defectors (with the fraction of cooperators in the community dwindling to zero, that is, $x_3 = x_4 = x_5 = \dots = 0$).

When $n \geq 2(m+1)$, we know, according to (3), that in generation 2 there must be at least one cooperator who is neighbored by $m/2$ cooperators to the left, and by $m/2$ cooperators to the right (receiving thereby the payoff of mR , see Figure 3). Consider then the neighborhood of a cooperator who is separated from the cluster of the $m+1$ defectors by exactly $m/2 - i$ cooperators, where $i = 1, \dots, m/2$. This cooperator receives a payoff of $(m-i)R + iS$. Given (1), and since $S < R$, $(m-i)R + iS < (m-(i-1))R + (i-1)S$, the information about a (cooperator) recipient of a payoff mR spreads to the descendents of the $m/2$ immediately neighboring less successful cooperators to the left and / or to the right (who receive payoffs of $(m-i)R + iS$, $i = 1, \dots, m/2$). Analogously, the information about the payoff of the most successful defector (who receives a payoff of $m/2(T+P)$) spreads to the descendents of the $m/2$ immediately neighboring less successful defectors (who receive payoffs of $(m/2 - i)T + (m/2 + i)P$, $i = 1, \dots, m/2$). Hence, the defector cluster neither expands nor shrinks; the fraction of cooperators in generation 3 is

$$x_3 = x_2. \quad (4)$$

In generation 4, the success of the cooperator who receives a payoff of mR and who is separated from the $m+1$ neighboring defectors by $m/2$ less successful cooperators (receiving a payoff of $(m-i)R + iS$, $i = 1, \dots, m/2$), is replicated by the less successful cooperators' descendents, and we infer that the fraction of cooperators remains constant at $x_4 = \dots = \hat{x} = (n-m-1)/n$, where \hat{x} represents the fixed point of the dynamics brought about by the difference equation $x_t = x_{t-1} = (n-m-1)/n$ for generation $t \geq 2$. For this reason, Figure 3 depicts also the equilibrium (or steady state) configuration in the wake of the mutation ($C \rightarrow D$) of a single individual. Therefore, under (1), we observe a (constant) positive fraction of cooperators in the community in the long run only if $n \geq 2(m+1)$.⁸

⁸ Investigating other initial configurations of defectors, we find that under (1), clusters of q mutant neighboring defectors where $2 \leq q \leq m/2$ expand to clusters of no more than $q + m$ defectors, as long as initially the clusters of size q are separated from each other by at least $m/2 + (m+1) + m/2 = 2m + 1$ neighboring cooperators. Clusters of at least $(m/2) + 1$ mutant neighboring defectors remain invariant (in terms of their initial size) as long as they are separated from each other by at least $m+1$ neighboring cooperators. Therefore, the "fate" of the community depends not only on the number of mutant defector clusters, but also on their spread, that is, on the space between them (cf. Stark and Behrens, 2010). If there are only a few isolated mutant defectors in a large community of cooperators then, by and large, the community remains a community of cooperators, the few spotted clusters of defectors notwithstanding. Then, defectors never "take over" the entire population. If, however, the number of mutations is excessive, there will not be enough space left between the mutants to avert the spread of the defection strategy over

This is a good point to say a word as to what will happen if the left hand side inequality in (1) does not hold and instead, $(m-1)R+S > \frac{m}{2}(T+P)$. In such a case, the second generation “response” to the appearance of a mutant defector in generation 1 - as captured by x_2 - will be as per (3). But in generation 3, the cluster of neighboring defectors will shrink from $m+1$ to $m-1$, if $n \geq 2(m+1)$, since the descendents of the defectors at the edge of the defector cluster will replicate the behavior of the cooperators receiving a payoff of $(m-1)R+S$ rather than the behavior of their fathers receiving a payoff of $\frac{m}{2}(T+P)$.⁹ Depending on the relative payoffs, this shrinkage will go on from one generation to the next until the farmers at the border of the remaining defector cluster will receive larger or equal payoffs than those of the cooperators $\frac{m}{2}$ farms away, or until only a single defector remains (a configuration that is identical to the configuration prevailing at the initial generation upon the appearance of a mutant defector). In both cases, in the next generation the size of the cluster of neighboring defectors will increase by m , and the process of shrinking will start all over again. This results in a farmer community where both cooperative and defection behavior prevail in the long run, with the fraction of cooperators periodically obtaining the same values (all different from zero and elements of an invariant set of at most m but not less than two elements). That is, the result is a fraction of cooperators, x_t , for $t \rightarrow \infty$, converging towards a periodic point with a prime period equal to or greater than 2. Therefore, also for $(m-1)R+S > \frac{m}{2}(T+P)$, cooperative behavior will be sustained within the farmer community in the long run if $n \geq 2(m+1)$.

These results guide us in calculating next the steady-state per capita payoff (per capita income) under (1), a measure of the social welfare of the community of n farmers, as a function of the span of interaction m .

the entire farming community: the requirement that for at least one *isolated* mutant defector there have to be at least $2m+1$ neighboring cooperators to the left and $2m+1$ neighboring cooperators to the right who separate the defector from other *isolated* defectors is the minimal requirement needed to guarantee the long-run survival of a positive fraction of cooperating individuals.

⁹ If, moreover, $(m-i)R+iS > (m/2-i-1)T+(m/2+i-1)P$, where $i \in \{2,3,\dots,m/2\}$ then, since farmers imitate the better of the strategies of their fathers and the m trading partners of their fathers, the size of the generation-3 defector cluster will be $m+1-2i$ (rather than $m-1$).

4 Can trading with more individuals decrease social welfare?

From the preceding discussion we infer that in the wake of the appearance of a mutant defector, for $T + P < 2R$, a community of initially cooperating, locally learning and optimizing individuals can eventually exhibit either heterogeneity or perfect homogeneity (consisting entirely of defectors), depending on the span of interaction, with heterogeneity being possible only if farmers trade with and learn from only a few (relatively “close”) individuals (that is only if $2 \leq m \leq \frac{1}{2}(n-2)$). Put differently, a persistent coexistence of cooperating behavior and defection behavior is possible only if the span of interaction is small, whereas a large number of trading partners leads to the extinction of cooperation. Given that $T + P < 2R$, once a single individual has mutated to become a defector, a reversion to a pure cooperator community cannot occur.

Furthermore, we have seen that a single defector in a community of cooperators can only be “successful” in the sense of “spreading the D -strategy over the entire community” if the community size, n , is small relative to the span of interaction, m . That is, after the information about the mutant defector’s mT payoff has spread (converting m descendents into additional defectors), there must be at least $m+1$ neighboring cooperators left - guaranteeing that one cooperator who is neighbored by $\frac{m}{2}$ farmers to the left, and by $\frac{m}{2}$ farmers to the right receives a payoff of mR - in order for cooperative behavior to be sustained. We can therefore conclude that if individuals trade with, and learn from more than $\frac{1}{2}(n-2)$ individuals, defection (springing from the appearance of a single mutant defector) will eventually spread over the entire community, whereas otherwise it will not; cooperating behavior will still prevail. What can we infer from this review of alternative configurations about the wellbeing of the community?

From section 3 we know that for $n \geq 2(m+1)$, which implies that the size of the community is at least six, the steady-state community, under (1), in the wake of the mutation of one cooperator farmer into a defector farmer consists of a cluster of $(m+1)$ defectors, with the remainder $n-m-1$ farmers being cooperators. Thus we know that we have one defector neighbored by $\frac{m}{2}$ defectors on each side, two defectors neighbored by $\frac{m}{2}$ defectors on one side and $\frac{m}{2}-1$ defectors and a cooperator on the other side, two defectors neighbored by $\frac{m}{2}$

defectors on one side and $m/2 - 2$ defectors and two cooperators on the other side, ..., two defectors neighbored by $m/2$ defectors on one side and one defector and $(m/2 - 1)$ cooperators on the other side, two defectors neighbored by $m/2$ defectors on one side and $m/2$ cooperators on the other side, two cooperators neighbored by $m/2$ cooperators on one side and $m/2$ defectors on the other side, two cooperators neighbored by $m/2$ cooperators on one side and a cooperator and $m/2 - 1$ defectors on the other side, ..., two cooperators neighbored by $m/2$ cooperators on one side and $m/2 - 1$ cooperators and a defector on the other side, and $n - (2m + 1)$ cooperators neighbored by $m/2$ cooperators on both sides. This configuration yields the following aggregate steady-state payoff for the n -farmer ($n \geq 2(m + 1)$) community (cf. Figure 3):

$$\begin{aligned}
 & mP + 2((m-1)P + T) + 2((m-2)P + 2T) + \dots + 2((m/2+1)P + (m/2-1)T) + m(P + T) \\
 & + m(R + S) + 2((m/2+1)R + (m/2-1)S) + \dots + 2((m-1)R + S) + (n - (2m+1))mR \\
 & = mP + 2 \sum_{i=1}^{m/2} [(m-i)P + iT] + 2 \sum_{i=1}^{m/2} [(m-i)R + iS] + (n - 2m - 1)mR \\
 & = mP + 2(P + R) \sum_{i=1}^{m/2} (m-i) + 2(S + T) \sum_{i=1}^{m/2} i + (n - 2m - 1)mR \\
 & = mP + \frac{1}{4}(3m-2)m(P + R) + \frac{1}{4}(m+2)m(S + T) + (n - 2m - 1)mR \\
 & = \frac{1}{4}(3m+2)mP + \frac{1}{4}(m+2)m(S + T) + (n - \frac{1}{4}(5m+6))mR.
 \end{aligned} \tag{5}$$

The per capita payoff for $2 \leq m \leq \frac{1}{2}(n-2)$ can be calculated from (5), and is depicted in the left five columns of Figure 4 for $n = 22$ where, without loss of generality, we normalize the payoffs (satisfying (1)) to range between 0 and 1 such that $S = 0$, $P = 0.15$, $R = 0.60$, and $T = 1$. The per capita payoff for $\frac{1}{2}(n-2) < m \leq n-2$, that is, for larger spans of interaction, changes quite dramatically to mP , as is shown in the right five columns of Figure 4. In sum, $Y(n, m)$, the per capita payoff, under (1), as a function of population size, n , and of the span of interaction, m , for $2 \leq m \leq n-2$, is given by

$$Y(n, m) = \begin{cases} \frac{1}{4} \frac{m}{n} [(3m+2)P + (m+2)(S+T) + (4n-5m-6)R] & 2 \leq m \leq \frac{1}{2}(n-2) \\ mP & \frac{1}{2}(n-2) < m \leq n-2. \end{cases} \tag{6}$$

When $2 \leq m \leq \frac{1}{2}(n-2)$, the community includes a cluster of $m+1$ defectors, and a complementary cluster of $m+1$ or more cooperators. The payoff of a single cooperator nested in the interior of the cooperator cluster (separated from the defector cluster by at least $m/2$ cooperators) is equal to mR , thus linearly increasing in the span of interaction, while the number of cooperators located within the long-run sustainable cooperator cluster decreases with an

increasing span of interaction. When $\frac{1}{2}(n-2) < m \leq n-2$, the per capita payoff is given by mP , thus again linearly increasing in the span of interaction (an inspection of (6) reveals likewise). These observations leave us with the question: when is it that the per capita payoff, under (1), obtains a maximum? At $\frac{1}{2}(n-2)$, or at $(n-2)$?

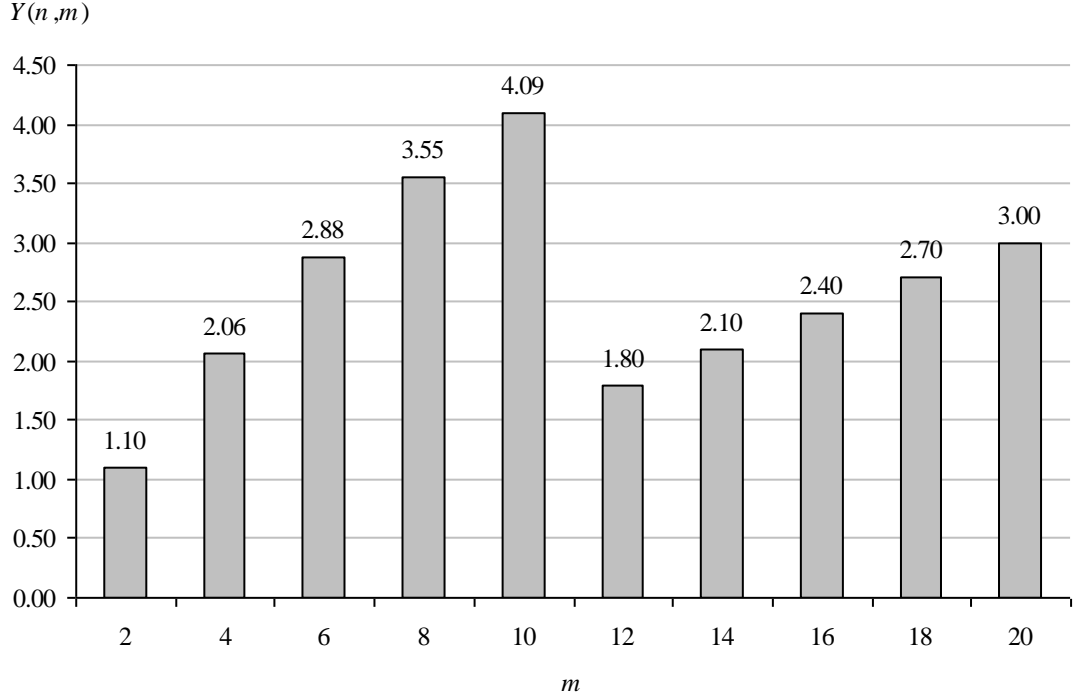


Figure 4: Per capita payoff as a function of the span of interaction, $2 \leq m \leq n-2$, for $n=22$ farmers, and when the payoffs are $S=0$, $P=0.15$, $R=0.60$, and $T=1$

Claim 1: Let $S = 0$, and let $\frac{1}{2}(n-2)$ be an even integer.¹⁰ Under condition (1) and $5P \leq T$, the per capita payoff $Y(n, m)$ is maximal at $m^* = \frac{1}{2}(n-2)$.

Proof: Given (1) and an even $\frac{1}{2}(n-2)$, the highest possible per capita payoff for $2 \leq m \leq \frac{1}{2}(n-2)$ is $Y(n, \frac{1}{2}(n-2))$. To see this, consider the term $[Y(n, m) - Y(n, m-1)]$ for $2 < m \leq \frac{1}{2}(n-2)$. We have that

$$\begin{aligned}
 & Y(n, m) - Y(n, m-1) \\
 &= \frac{1}{4} \frac{m}{n} [(3m+2)P + (m+2)(S+T) + (4n-5m-6)R] \\
 &\quad - \frac{1}{4} \frac{(m-1)}{n} [(3(m-1)+2)P + ((m-1)+2)(S+T) + (4n-5(m-1)-6)R] \\
 &= \frac{1}{4n} [(3P+S+T-5R)m^2 + 2(P+S+T+(2n-3)R)m] \\
 &\quad - \frac{1}{4n} [(3P+S+T-5R)(m-1)^2 + 2(P+S+T+(2n-3)R)(m-1)] \\
 &= \frac{1}{4n} [(3P+S+T-5R)(2m-1) + 2(P+S+T+(2n-3)R)],
 \end{aligned} \tag{7}$$

¹⁰ If $\frac{1}{2}(n-2)$ were an odd number, the condition $5P \leq T$ would have to be replaced by $7P \leq T$.

and that

$$\begin{aligned}
 & Y(n, m) - Y(n, m-1) \\
 & > \frac{1}{4n} [(3P + S - 4R)(2m-1) + 2(P + S + T + 2(2m+2)R - 3R)] \\
 & = \frac{1}{4n} [(3P + S)(2m-1) + 2(P + S + T) + (2m+2)4R - 6R - (2m-1)4R] \\
 & = \frac{1}{4n} [\underbrace{(3P + S)}_{=3P}(2m-1) + 2\underbrace{(P + S + T)}_{=P+T} + 6R] \\
 & = \frac{1}{4n} [(6m-1)P + 2T + 6R] > 0,
 \end{aligned} \tag{8}$$

where the inequality in the second line of (8) is due to $R < T$ and $2 \leq m \leq \frac{1}{2}(n-2)$, as the latter is equivalent to $6 \leq 2(m+1) \leq n$. Thus, for $2 \leq m \leq \frac{1}{2}(n-2)$ and an even $\frac{1}{2}(n-2)$, the highest possible per capita payoff is obtained for $m = \frac{1}{2}(n-2)$. When, $\frac{1}{2}(n-2) < m \leq n-2$, the per capita payoff is equal to mP (cf. (6)), in which case the highest possible per capita payoff is obtained for $m = n-2$. Moreover, we know that $S = 0$ in combination with $5P \leq T$ and with $\frac{m}{2}(T + P) < mR$ (the right hand side inequality of (1)) imply that $3P < R$: a cooperator trading with a cooperator receives a payoff at least three times that of a defector trading with a defector. Using this result, we find that for an even $\frac{1}{2}(n-2)$

$$\begin{aligned}
 & Y(n, \frac{1}{2}(n-2)) \\
 & = \frac{1}{4} \frac{(n-2)}{2n} [(3(\frac{n}{2}-1) + 2)P + (\frac{n}{2}-1 + 2)(S + T) + (4n - 5(\frac{n}{2}-1) - 6)R] \\
 & = (n-2) \frac{1}{16n} [(3n-2) \underbrace{(P+R)}_{>4P} + (n+2) \underbrace{(S+T)}_{\geq 5P}] \\
 & > (n-2) \frac{1}{16n} [(12n-8)P + (5n+10)P] \\
 & = (n-2)P + \frac{1}{16n} (n^2 - 4)P \\
 & > (n-2)P \\
 & = Y(n, n-2).
 \end{aligned} \tag{9}$$

Therefore, for an even $\frac{1}{2}(n-2)$, the per capita payoff $Y(n, m)$ is highest at $m = \frac{1}{2}(n-2)$.

■

From Claim 1 it follows that even though one might well expect the community's per capita payoff to be positively correlated with the span of trade (that is, with the number of trading partners given by m), a setting (characterized by (1), $S = 0$, and $5P \leq T$) exists for which this is not the case: for a sufficiently high "temptation to defect," where "sufficiently high" is rendered here by the defector-cooperator payoff being higher by five times or more than the defector-defector payoff, per capita payoff is at its peak when the span of interaction obtains its highest possible "local" value. The intuition is that when individuals' optimization is based on the assessment of *less* information (corresponding to what could be described as information being spanned *locally*, that is, $2 \leq m \leq \frac{1}{2}(n-2)$), the seemingly good news about the payoff of a

cooperator mutating into a defector does not reach some clusters of cooperators who therefore remain immune to the influence of the mutant defector. Since per capita payoff grows monotonically (cf. the proof of Claim 1), the highest per capita payoff is obtained for $m = \frac{1}{2}(n-2)$ that is, for the largest span of information (which, to recall, in this model is identical to the span of interaction) is still “localized.” Figure 4 shows this result for a community of 22 farmers: starting from $m = 2$, we see that as the span of trade m increases, the steady-state per capita payoff rises to its global maximal level of $Y(22,10) = \frac{5}{88}[64(P+R) + 24(S+T)] \cong 4.09$, and thereafter (for $10 = \frac{1}{2}(n-2) < m \leq n-2 = 20$) it falls sharply to its “lower branch,” where it is given by mP , which reaches a local maximum at $(n-2)P = 20P = 3$.

We thus infer that as long as the span of interaction is not large, that is, as long as it is sufficiently smaller than $m^* = \frac{1}{2}(n-2)$, trading with more individuals can increase social welfare. In other words, when individuals trade with relatively few individuals (corresponding to what could be described as local trade, associated with $2 \leq m \leq \frac{1}{2}(n-2)$), the social outcome arising from an expansion of trade can be superior to that which would have been obtained had the same expansion of trade taken place with the individuals engaging in trade with more individuals (corresponding to what could be described as global trade, associated with $\frac{1}{2}(n-2) < m \leq n-2$). When at first the trade is with a few, trading with a few more could confer a welfare gain; when at first the trade is with fairly many, trading with a few more might reduce welfare.

5 Conclusions

Given that individuals who live along a circle (a lake) learn from those with whom their fathers traded, we can also learn a lesson or two from a study of their experience.

First, when the span of interaction widens, a mutant defector does not convert the farmer community into a community that uniformly replicates his conduct, as long as sufficiently sized islands of cooperators remain immune to his example. Widening the span of interaction locally is then not a threat to the long-run coexistence of cooperators and defectors.

Second, in this paper we use an information environment that coincides with the structure of exchange. This differs from Stark and Behrens (2010), where it has been shown that gathering information from distant people rather than relative to gathering information from people nearby allows cooperation to spread widely within a community - but only up to a certain point: “global” information results in the extinction of cooperation and in a reduction of welfare. Our current finding is that more trade does not necessarily increase social welfare. Moreover, in Stark and Behrens (2010) it is assumed that while individuals trade only with their adjacent neighbors, information can be harvested from farther afield. Here, however, the number of trading partners, m , increases beyond adjacent neighbors. Nonetheless, in a population of size n , playing $n-2$ prisoner dilemma games can easily yield a smaller per capita payoff than playing $\frac{1}{2}(n-2)$ prisoner dilemma games. From this observation we conclude that trading with many more (as opposed to trading with a few more) may not bring in its wake higher social welfare. Put differently, while the expansion of the market can be desirable under some circumstances, it might not be beneficial under others; going global may not be a recipe for welfare maximization.

In future work we could develop the model in several interesting directions. For example, we could explore generalizations to network structures other than a circle (such as a star, a crystal, or complete networks). In such contexts, we will seek to identify the link between the network structures and clustering¹¹ and explore whether a systematic relationship exists between, on the one hand, the difference between the span of information and the span of interaction and, on the other hand, social welfare outcomes.¹²

¹¹ See, for example, Ohtsuki et al. (2006), or Jun and Sethi (2007).

¹² See, for example, Hoffmann (1999) for a simulation-based discussion of the effect of local learning and local interaction on the prevalence of cooperation.

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