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A Theory of Migration as a Response to Relative Deprivation

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Abstract

We model migration as a response to relative deprivation. We present a specific configuration of incomes in which the process of migration in response to relative deprivation reaches a steady state. However, for the general configuration of incomes we show that it is impossible to prove the existence of a steady state. We study the social welfare implications of the two cases and show that if individuals are left to pursue their betterment, the resulting state falls short of the best social outcome. We present several implications of the model including federalism and the demand for secession.

Zusammenfassung

Wir modellieren Migration als eine Reaktion auf relative Verarmung. Wir stellen eine spezifische Einkommenskonfiguration vor, in der der Migrationsprozess in Reaktion auf relative Verarmung ein konstantes Gleichgewicht erreicht. Für die allgemeine Einkommenskonfiguration zeigen wir jedoch, dass es unmöglich ist, die Existenz eines konstanten Gleichgewichts nachzuweisen. Wir untersuchen die Wohlfahrtsimplikationen der beiden Fälle und zeigen, dass, wenn es den Individuen überlassen bleibt, die Verbesserung ihrer Lebensumstände herbeizuführen, das Resultat hinter dem besten sozialen Ergebnis zurückbleibt. Wir stellen verschiedene Implikationen des Modells vor, einschließlich des Föderalismus und der Sezessionsforderung.

1   Introduction

Discontent can arise not only from having a low wage but also from having a wage that is lower than that of others. Given the set of individuals with whom comparisons are made, an unfavorable comparison could induce harder work. This idea is captured and developed in the literature on performance incentives in career games and other contests. (Early studies include Lazear and Rosen (1981), Rosen (1986), and Stark (1990).)

An unfavorable comparison could also induce a departure for work elsewhere, where wages are higher, *without* changing the set of individuals with whom comparisons are made. This response is taken up in the literature on relative deprivation and migration. Earlier studies include Stark (1984), Stark and Yitzhaki (1988), and Stark and Taylor (1989,1991). Two results from our earlier studies merit mention here: First, drawing largely on the work of social psychologists, especially Runciman (1966), a set of axioms was formulated and several propositions were stated and proved. The key idea of the earlier studies is that a comparison of the income of *i* (an individual, a household, a family) with the incomes of others who are richer in *i*'s reference group results in *i*'s feeling of relative deprivation. The associated negative utility impinges on migration behavior. In particular, we have shown that the relative deprivation of an individual (or, for that matter, of a household or a family) whose income is *y* is

$$RD(y) = \int_{y}^{\infty} [1 - F(x)] dx$$

where $F(x)$ is the cumulative distribution of income in *y*'s reference group.[1] Using some algebra we have further shown that $RD(y) = [1 - F(y)] \cdot E(x - y / x > y)$: The relative deprivation of an individual whose income is *y* is equal to the proportion of those in *y*'s reference group who are richer than *y*, times their mean excess income. Second, a distaste for relative deprivation matters; relative deprivation is a significant explanatory variable of migration behavior.

Yet a third response could be to sever ties with the offensive set. Leaving to associate with another set, without a change in one's income, could also dampen relative deprivation. This reaction, not studied in earlier work on migration and relative deprivation, is the subject of the present paper. Holding income constant (as if the individual is born with an income) enables us to study migration behavior that is purely due to relative deprivation. The present paper can thus be conceived as the dual of earlier work: while in past work the reference group was held constant and migration with a gain in income served to reduce relative deprivation within a given

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1   The analogous definition of RD for a finite discrete set of individuals whose incomes are $y_1,\ldots,y_n$, where

$y_1 ™ y_2 ™ \ldots ™ y_n$, is $RD(y_i) = \sum_{j=i}^{n-1} [1 - P(y_j)] (y_{j+1} - y_j)$ for i = 1, ..., n where $P(y_j) = \text{Prob}(y \le y_j)$.

reference group, the present paper holds income constant and relative deprivation is reduced through migration with a substitution of reference groups. The main questions are: Does the process of migration in response to relative deprivation reach a steady state (wherein all migration ceases and no one is able to reduce his relative deprivation through migration)? Does migration by individuals in response to their aversion to relative deprivation lower societal relative deprivation? Does it minimize societal relative deprivation? We study the relationship between relative deprivation and migration by considering two examples. We list several results suggested by the examples. In conclusion, we offer conjectures illustrated by our examples, and reflect on several issues that pertain to the distaste for relative deprivation.

2 The Basic Setup

Suppose there are two regions, *A*, and *B*, and that an individual's relative deprivation arises only from comparisons with other individuals in his region; nothing else matters. We abstract from the intrinsic value of *x*. However, this is of no consequence whatsoever since *x* is retained (the individual's income is held constant) across regions. We are thus able to study migration behavior that is purely due to relative deprivation. The individual prefers to be in the region where his relative deprivation is lower. The individual does not care about the regions themselves. When equally relatively deprived (a tie), the individual does not migrate. The individual cannot take into account the fact that other individuals behave in a similar fashion. However, the individual's payoff, or utility, depends on the actions of all the individuals regardless of whether their incomes are higher or lower than his.[2] A key feature of this situation is that tomorrow's migration behavior of every individual is his best reply to today's migratory actions of other individuals. What will the migration path and the associated behavior look like? Will there be a steady-state distribution of the individuals across the two regions?[3]

2.1 Example 1

Suppose there are *n* individuals and that individual *i* receives income *i*. Thus the configuration of incomes is $(1,...,n-1,n)$. Suppose that initially all the individuals $1,...,n-1,n$ are in region *A*. Region *B* opens up. (For example, migration restrictions are eliminated, or *B* comes into existence.) We measure time discretely.

Claim 1: If the configuration of incomes is $(1,...,n-1,n)$, then the process of migration in response to relative deprivation reaches a steady state in just one period. Moreover, at the steady state, the individual with income *n* remains in region *A* while the rest of the population stays in region *B*.

Proof: It is trivial that in period 1 the individual with income *n* stays in region *A* while the rest of the population migrates to region *B*. Now consider the action of the individual with

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2 In particular, the departure of a low-income individual raises the relative deprivation of higher-income individuals. This occurs because the weight these individuals attach to the difference between the incomes of individuals richer than themselves and their own income must rise.

3 The individual is rational but not sophisticated; in the parlance of (evolutionary) game theory, the individual is "naive" – he does not understand or believe that other individuals are similarly adjusting their location nor that his migratory behavior potentially affects the migratory behavior of other individuals. We ask whether a population of individuals who are "myopically groping" toward minimizing their relative deprivation will reach a steady state, and we seek to specify the discrete time dynamics – the inter-regional distribution of the population in period *t*+1 as a function of the inter-regional distribution of the population in period *t*.

income i, where $i = 1,...,n-1$. If the individual remains in region B, the individual's relative deprivation will be $\frac{(n-i)(n-1-i)}{2(n-1)}$; if the individual returns to A, the individual's relative deprivation will be $\frac{n-i}{2}$. Note that $\frac{(n-i)(n-1-i)}{2(n-1)} < \frac{n-i}{2}$ for $i = 1,...,n-1$. We thus have the result of the Claim. $\square$

Corollary: Given the above setup and a real number $\alpha > 0$, the process of migration in response to relative deprivation will be identical in the two populations $P = \{1,...,n-1,n\}$ and $P_\alpha = \{\alpha,...,\alpha(n-1),\alpha n\}$.

Proof: The proof of the Corollary is a replication of the proof of Claim1 since the two measures of relative deprivation in the proof of Claim 1 are multiplied by α, and therefore the inequality in the proof of Claim 1 carries through to the case of the Corollary. $\square$

It follows that the propensity prompted by relative deprivation to engage in migration by a rich population is equal to the propensity prompted by relative deprivation to engage in migration by a uniformly poorer population. The pattern of migration is independent of the general level of wealth of the population.[4]

Note that the steady state is independent of whether individuals migrate simultaneously (as assumed) or in the order of their relative deprivation (with the most relatively deprived migrating first, the second most relatively deprived migrating second, and so on). In the latter case the steady state is reached after *n*-1 periods rather than in just one period.

Each of the two groups that form in the steady state is smaller than the original single group. It might therefore be suspected that migration is caused partly or wholly by an aversion to crowding. It is easy to see, however, that this is not so. When 1000 individuals, each with income y, are in A there is crowding but no migration; when 10 individuals, 5 with income y>1 each and 5 with income y-1 each are in A there is little crowding but much migration.

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4 Note that the results of this section go through even if the population is multiplied by a natural number k. To see this consider the configuration of incomes $\left(\underbrace{1,...,1}_{k},...,\underbrace{n,...,n}_{k}\right)$. In period 1 the k individuals with income n stay in region A while the rest of the population migrates to region B. Now consider the action of an individual with income i, where $i = 1,...,n-1$. If the individual remains in region B, the individual's relative deprivation will be $\frac{(n-i)(n-1-i)}{2(n-1)}$ (as when $k=1$). If an individual with income i were to return to A, the individual's relative deprivation would be $\frac{k}{k+1}(n-i)$. Since for any natural number k, $\frac{k}{k+1}(n-i) > \frac{(n-i)(n-1-i)}{2(n-1)}$, the result of Claim 1 holds also for the case in which the population is multiplied by k.

2.2   Example 2

That a steady state is reached when incomes are equally spaced is not however a result that carries through to the general configuration of incomes. We next show that there exists a configuration of non-equally spaced incomes for which a steady state is not reached.

Claim 2: If the configuration of incomes is $(1, 5 - \varepsilon - \delta, 5 - \varepsilon, 5)$, where $0 < 2\varepsilon < \delta < 4/5$, then the process of migration in response to relative deprivation fails to reach a steady state.

Proof: By directly calculating the relative deprivations of the individuals constituting the population, we can easily trace the migratory moves of the individuals in periods 0 through 5, which are presented in Figure 1.

Figure1: The Migration Process of the Population of Claim 2 in Response to Relative Deprivation in Periods 0 through 5

Period 0	
Region A	Region B
5	
5 - ε	
5 - ε - δ	
1	

Period 1	
Region A	Region B
5	
	5 - ε
	5 - ε - δ
1	

Period 2	
Region A	Region B
5	
	5 - ε
	5 - ε - δ
1	

Period 3	
Region A	Region B
5	
	5 - ε
5 - ε - δ	
1	

Period 4	
Region A	Region B
5	
	5 - ε
5 - ε - δ	
1	

Period 5	
Region A	Region B
5	
	5 - ε
	5 - ε - δ
1	

Note that the population distribution in period 5 is the same as the population distribution in period 1. This implies that the process of migration in response to relative deprivation fails to reach a steady state.[5] □

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5 Suppose we endow individuals with "sophisticated reasoning ability." The individual takes into account the fact that other individuals are avoiding relative deprivation, such that today's migration behavior is the individual's best reply to tomorrow's migration by others. This contemplation ability is sufficient to break the cyclicity in this example and achieve a steady state in just one period. The reasoning leading to this outcome is as follows. Take $\varepsilon = 0.2$ and $\delta = 0.5$. The configuration of incomes is then 5, 4.8, 4.3, 1. It is trivial that 5 will stay in A and that 4.8 will migrate to B. The only "real" contemplation is of 1 and 4.3. It is clear that 4.3 will prefer to be with 1 because in a three-individuals' configuration the weight that 4.3 will attach to the quite similar incomes 5 or 4.8 is 1/3 rather than 1/2. Being aware of 4.3's reasoning (and realizing that 4.3 will be "glued" to him), 1 figures out that he will be better off with 4.8 rather than with 5. Thus, 1's preferred location is B. It follows then that in period 1 individuals 4.8, 4.3, and 1 will be in B. Figure 2 diagrammatically presents this outcome.

Figure 2: The Migration Process of the Population of Claim 2 in Response to Relative Deprivation under "Sophisticated Reasoning Ability," and when $\varepsilon = 0.2$ and $\delta = 0.5$

	Period 0		Period 1	
	Region A	Region B	Region A	Region B
	5		5	
	4.8			4.8
	4.3			4.3
	1			1

Apparently, the outcome of non-convergence to a steady state is not robust to variation in the sophistication of individuals' reasoning ability.

3 Societal Relative Deprivation

The examples alluded to in Section 2 point to an additional result that pertains to social welfare. Suppose we measure social welfare by the inverse of the population's total relative deprivation, where total relative deprivation is the sum of the relative deprivations of all the individuals constituting the population. It follows that social welfare is maximized when total relative deprivation is minimized. In both examples migration raises social welfare since at the initial period 0 total relative deprivation is maximal. Yet individualistic behavior fails to produce the best social allocation. Consider first Example 1. The steady-state allocation has n in region A and $(n-1,...,1)$ in region B. This allocation is Pareto efficient. However, the minimal total relative deprivation (TRD) obtains when $(n, n-1,...,i)$ are in region A and $(i-1, i-2,...,1)$ are in region B where $i = \dfrac{n}{2} + 1$ if n is an even number, and where $i = \dfrac{n+1}{2}$ or $i = \dfrac{n+3}{2}$ when n is an odd number.[6] Consider next Example 2. The minimal period j, $j=0,...,5$, TRD is $2 + \delta/2$, obtained either at period 2 or, as it so happens, at period 4. However, the minimal TRD is $\dfrac{2}{3}(\varepsilon + \delta)$; it is obtained when 5, $5 - \varepsilon$, and $5 - \varepsilon - \delta$ are in region A, and 1 is in region B. As in Example 1, the minimal TRD is rendered by an allocation that is not attained through individualistic behavior.

Left to pursue their individualistic betterment, individuals end up at a state that falls short of the best social outcome. This is not surprising. We should not expect that the distribution of individuals across the two regions will represent a welfare maximizing or a collectively preferred division when no individual pays any heed to the relative deprivation he inflicts upon members of the group he joins or upon members of the group he leaves. This failure suggests a role for social planners or the government to distribute the population across the two regions to attain the social optimum.

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6 The proof is in the Appendix.

4 Conclusions and Complementary Reflections

The opening of another region, *B*, facilitates shedding of relative deprivation. Consider a reverse process, wherein regions *A* and *B* merge into a single composite region that constitutes everyone's reference group. In all cases (except the degenerate case in which all individuals have exactly the same income) the population's relative deprivation is bound to rise. Groups who are less well off in terms of absolute income will be better off in terms of wellbeing if they are allowed to secede, without any change in absolute income. Conversely, a group that is less well off in terms of absolute income that is forced to merge with a group that is better off in terms of absolute income becomes worse off. The pressure to form a separate state, for example, can be partially attributed to this aversion to relative deprivation; when such an aversion exists, the sole individual with less than 1 in *B* may prefer that option to having 1 in *A*, where 2 is present.

These considerations relate to federalism. The process of adding new members to a federation of nations usually draws on the expectation that in the wake of the integration the incomes of the citizens of the new member nations will rise. The European Union, however, has taken great pains to ensure that the incomes of the citizens of the would-be member nations rise substantially *prior* to integration. Our approach suggests a rationale. To the extent that integration entails the formation of a new reference group, relative deprivation when 1 joins 2 would be reduced if 1½ were to join 2, and would be eliminated altogether if 2 were to join 2.

There are other spheres in which a distaste for relative deprivation and migration behavior are intertwined. The often observed clustering of migrants at destination can be attributed to a choice that migrants make rather than to a constraint to which they yield. This clustering was shown to confer an informational edge to migrants – lower recognition costs – in an environment characterized by incomplete information about the traits of partners to trade (Stark, 1999). Clustering may, however, also arise from a desire to reduce relative deprivation; the clustering together at destination intensifies contacts and cross references between migrants, effectively creating a reference group within which comparisons result in migrants sensing less relative deprivation than they would have been exposed to had they been scattered thinly throughout the receiving population.

We have assumed a given and uniform dislike of relative deprivation. Relative deprivation is a sensitive measure that encompasses rank-related information beyond mere rank. (It tells us that 1 compared to 3 is worse than 1 compared to 2, even though in both instances 1's rank is second.) An important question that is not addressed in this paper is where the aversion to relative deprivation or, for that matter, the distaste for low rank originates. Postlewaite (1998) argues that since over the millennia high rank conferred an evolutionary advantage in the

competition over food and mating opportunities, the concern for rank is likely to be hardwired (part of the genetic structure). More generally though, any setting in which rank impinges positively, directly or indirectly, on consumption ought to imply a concern for rank.[7] The study of why an aversion to relative deprivation exists and why individuals exhibit distaste for low rank invites more attention.

It is plausible to stipulate that the distaste for low rank will not be uniform across societies. Consequently, the extent of segregation across societies will vary. Since segregation is visible whereas preferences are not, an inference may be drawn from the observed segregation to the motivating distaste, with more segregation suggesting stronger distaste.

There is a long-held view that when it comes to rank (or social status), "one person's gain is another's loss" (Weiss and Fershtman, 1998) This is not necessarily true. Consider the example in which (1,4) are in A and (2,3) are in B, and the summary statistic of the rank-related information is, again, our measure of relative deprivation. 1 seeks to reduce his relative deprivation, and therefore migrates to B, where his relative deprivation is 1 rather than $3/2$ in A. 1's arrival at B also *lowers* 2's relative deprivation from $1/2$ prior to 1's arrival to $1/3$ following his arrival. Since the relative deprivations of 4 and 3 remain intact, at zero, one person's gain is another's *gain*: although the entrant into B does not internalize the change in the relative deprivations of the "incumbents," the resulting steady-state allocation is nonetheless Pareto efficient.

In future work we could extend the analysis of the present paper by inquiring how individuals behave when migration is costly. Suppose there is a fixed cost, $c > 0$, of migration from one region to another. If migration were to occur it would entail a reduction of income, thus violating the present paper's assumption that income is held constant. To incorporate the presence of a migration cost it will then be necessary to employ a measure of utility that weighs in both relative deprivation and absolute income. (Migration will entail a trade-off between relative deprivation and absolute income.) In such an environment, if individuals were to continue to migrate (if migration does not eventually cease), the cumulative costs of switching may add up to a large-scale welfare loss. Hence, the conclusion of section 3 – advocating interference by the government in distributing the population across the two regions to attain the social optimum – will only be reinforced.

Note, however, that if we were to introduce the mild assumption that any combination of a positive income and relative deprivation is preferable to a combination of a zero (or a negative) income and any relative deprivation, no individual will engage in migration if $c \geq x_{n-1}$ (that is, if the cost of migration is at least as high as the income of the individual with the second highest income). In such a case, the social cost of the cost of migration is easily quantifiable: It is the

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7   In poor societies with meager assets, rank can serve as a proxy for collateral, thereby facilitating the attainment of credit.

difference between the inverse of the TRD that would have been achieved had migration been costless and the inverse of the initial TRD.

Finally, looking beyond migration, aversion to relative deprivation may impinge on the formation and dissolution of groups, clubs, neighborhoods, and other associations.

Appendix

To find the division of a group of *n* individuals across regions *A* and *B* that confers the minimal total relative deprivation (TRD) we proceed in two steps. First given the size of the two subgroups, we show that the minimal TRD is reached when high income individuals are in one of the subgroups and low income individuals are in the other subgroup. (That is, the income of *any* individual who is in one subgroup is higher than the income of *any* individual who is in the other subgroup.) Second, given this distribution, we show that the minimal TRD is reached when *half* of the individuals are in one subgroup and the other half are in the other subgroup.

Lemma: Let *n* be a fixed positive integer. Consider $\{a_1, a_2, ..., a_n\}$ where $a_1 < a_2 < ... < a_n$ and a_i's are positive integers. Let $S(a_1, a_2, ..., a_n) = \sum_{1 \le i, j \le n} |a_i - a_j|$. Then $S(a_1, a_2, ..., a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1, 2, ..., n-1$.

Proof: For any $i < j$, we have $|a_i - a_j| = |a_j - a_{j-1}| + |a_{j-1} - a_{j-2}| + ... + |a_{i+1} - a_i|$. Therefore, $|a_i - a_j| \ge j - i$ and $\left(\begin{array}{c} |a_i - a_j| = j - i \\ \text{for all i, j} \end{array}\right)$ if and only if $\left(\begin{array}{c} a_{i+1} = a_i + 1 \\ \text{for } i = 1, 2, ..., n\text{-}1 \end{array}\right)$. It follows that $S(a_1, a_2, ..., a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1, 2, ..., n-1$. (This minimum is $\dfrac{n(n^2 - 1)}{3}$.)

Corollary: Consider the configuration of incomes $(1, ..., n-1, n)$. Let there be two regions, *A* and *B*, with $(i_1, i_2, ..., i_{n_A})$ in *A*, and $(j_1, j_2, ..., j_{n_B})$ in *B*, $n = n_A + n_B$. Let $TRD = TRD_A + TRD_B$. Then, if n, n_A, n_B are fixed, *TRD* reaches its minimum if and only if $(j_1, j_2, ..., j_{n_B}) = (1, 2, ..., n_B)$ or $(i_1, i_2, ..., i_{n_A}) = (1, 2, ..., n_A)$; that is, either

Region A	Region B
n	
$\vdots$	
$n_B +1$	
	n_B
	$\vdots$
	1

or

Region A	Region B
	n
	$\vdots \quad n_A +1$
n_A	
$\vdots$	
1	

Proof: Note that $TRD_A = \dfrac{1}{2n_A} S(i_1, i_2, ..., i_{n_A})$, $TRD_B = \dfrac{1}{2n_B} S(j_1, j_2, ..., j_{n_B})$. Thus, for fixed n_A, n_B, $\min TRD_A \Leftrightarrow \min S(i_1, i_2, ..., i_{n_A})$, $\min TRD_B \Leftrightarrow \min S(j_1, j_2, ..., j_{n_B})$. Assume that TRD reaches its minimum at $(i_1^*, i_2^*, ..., i_{n_A}^*)$, $(j_1^*, j_2^*, ..., j_{n_B}^*)$. Without loss of generality, assume that $n \in (i_1^*, i_2^*, ..., i_{n_A}^*)$. Then, if $(i_1^*, i_2^*, ..., i_{n_A}^*) \neq (n_B +1, ..., n)$, then $(j_1^*, j_2^*, ..., j_{n_B}^*) \neq (1, ..., n_B)$. By the *Lemma*, we have that $TRD_A(i_1^*, i_2^*, ..., i_{n_A}^*) > TRD_A(n_B +1, ..., n)$, and $TRD_B(j_1^*, j_2^*, ..., j_{n_B}^*) > TRD_B(1, ..., n_B)$. Thus,

$TRD((i_1^*, i_2^*, ..., i_{n_A}^*), (j_1^*, j_2^*, ..., j_{n_B}^*)) > TRD((n_B +1, ..., n), (1, ..., n_B))$, which contradicts the assumption that TRD reaches its minimum at $(i_1^*, i_2^*, ..., i_{n_A}^*)$, $(j_1^*, j_2^*, ..., j_{n_B}^*)$. Hence, $(i_1^*, i_2^*, ..., i_{n_A}^*) = (n_B +1, ..., n)$, and $(j_1^*, j_2^*, ..., j_{n_B}^*) = (1, ..., n_B)$. Conversely, by the *Lemma*, we have that $TRD_A(i_1, i_2, ..., i_{n_A}) \geq TRD_A(n_B +1, ..., n)(or\,(1, ..., n_A))$, and $TRD_B(j_1, j_2, ..., j_{n_B}) \geq TRD_B(1, 2, ..., n_B)(or\,(n_A +1, ..., n))$. Therefore, TRD reaches its minimum at either of the two configurations. We thus proved the Corollary. $\square$

We next determine the size of the subgroups that brings *TRD* to a minimum.

Let $(n,...,i)$ be in region A, and let $(i-1,...,1)$ be in region B. Total relative deprivation in A is[8]:

$$TRD_A = \frac{1}{n-i+1} \cdot 1 + \frac{2}{n-i+1} \frac{1+2}{2} + ... + \frac{n\text{-}i}{n\text{-}i+1} \frac{1+2+...+n-i}{n-i}$$

$$= \frac{1+(1+2)+...+(1+2+...+n-i)}{n-i+1} = \frac{(n-i)(n-i+2)}{6} \quad .$$

Total relative deprivation in B is:

$$TRD_B = \frac{1}{i-1} + \frac{2}{i-1} \frac{1+2}{2} + ... + \frac{i-1-1}{i-1} \frac{1+2+...+i-1-1}{i-1-1}$$

$$= \frac{1+(1+2)+...+(1+2+...+i-2)}{i-1} = \frac{i(i-2)}{6} \quad .$$

Hence, $TRD = TRD_A + TRD_B = \frac{1}{6}[(n-i)(n-i+2)+i(i-2)]$. We seek to solve $\min_{1\le i\le n} TRD$. Since

$\frac{dTRD}{di} = \frac{1}{3}(-n+2i-2)$ and $\frac{d^2 TRD}{di^2} = \frac{2}{3} > 0$, we have that the minimal TRD obtains when

$\frac{dTRD}{di} = 0 \Rightarrow -n+2i-2 = 0 \Rightarrow i = \frac{n}{2}+1$. If n is an even number then the i that brings TRD to a

minimum is $i^* = \frac{n}{2}+1$, and, by direct calculation, $TRD = \frac{1}{12}(n^2-4)$. If n is an odd number,

direct calculation yields that when $i = \frac{n+1}{2}$, $TRD = \frac{1}{12}(n^2-3)$, and that when $i = \frac{n+3}{2}$,

$TRD = \frac{1}{12}(n^2-3)$. Therefore, if n is an odd number, the i that brings TRD to a minimum is

$i^* = \frac{n+1}{2}$ or $i^* = \frac{n+3}{2}$.

The result pertaining to the optimal split of the n individuals between the two regions can

also be obtained by noting that for $(1,2,...,n)$, $TRD = \frac{n^2-1}{6}$. (This equation can be inferred, for

example, from the expression above of $TRD_B = \frac{i(i-2)}{6}$ by setting $i-1 = n$.)

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8   $\sum_{k=1}^{n}(1+2+...+k) = \sum_{k=1}^{n}\frac{(1+k)k}{2} = \frac{1}{2}\sum_{k=1}^{n}k + \frac{1}{2}\sum_{k=1}^{n}k^2 = \frac{1}{2}\frac{(1+n)n}{2} + \frac{1}{2}\frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(n+2)}{6}$.

    Substituting $n-i$ for n yields the last expression of TRD_A.

Let $\quad n = n_A + n_B, \quad n \geq 2, \ n_A \geq 1$. Then $\quad TRD_A = \dfrac{n_A^2 - 1}{6} \quad$ and $\quad TRD_B = \dfrac{(n - n_A)^2 - 1}{6}$.

Therefore, $TRD = \dfrac{2n_A^2 + n^2 - 2n \cdot n_A - 2}{6}$. We seek to solve $\min\limits_{1 \leq n_A \leq n} TRD$. Since $\dfrac{dTRD}{dn_A} = \dfrac{4n_A - 2n}{6}$

and $\quad \dfrac{d^2 TRD}{d^2 n_A} = \dfrac{4}{6} > 0$, we have that the minimal TRD obtains when

$\dfrac{dTRD}{dn_A} = 0 \Rightarrow 4n_A - 2n = 0 \Rightarrow n_A = \dfrac{n}{2}$. Therefore, if n is an even number, half of the n individuals

will be in each of the two regions. With $\quad TRD_A = \dfrac{\left(\dfrac{n}{2}\right)^2 - 1}{6} \quad$ and $\quad TRD_B = \dfrac{\left(\dfrac{n}{2}\right)^2 - 1}{6}$,

$$TRD = TRD_A + TRD_B = 2 \dfrac{\left(\dfrac{n}{2}\right)^2 - 1}{6} = \dfrac{1}{12}\left(n^2 - 4\right).$$

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